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LETTER

Mode-I fracture toughness measurement of PMMA with the Brazilian disk test

Jun Zhou · Yang Wang · Yuanming Xia

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PMMA (Polymethyl methacrylate) is employed as an industrial material due to its excellent properties. Therefore, investigating the fracture behavior for this material is of importance in its structural applications. It is well known that fracture toughness is an important material parameter in characterizing mechanical properties of materials. So far, a number of experimental techniques have been developed for determining the fracture toughness of materials under different conditions [1]. Particularly, Brazilian disk (central cracked circular disk) test technique is widely used to measure the fracture toughness of brittle materials under mixed-mode loading conditions [2–4]. The schematic diagram of Brazilian disk and loading configuration is shown in Fig. 1. The diameter of the disk is 2R and the crack length is 2a. A pair of compressive load, P, is applied at the boundary and across the diameter of the disk. The angle of inclination of the crack plane relative to the line of loading is defined as loading angle θ. The advantage of the Brazilian disk geometry is that by changing the angle θ the entire range of crack-tip mode mixites from pure mode I to pure mode II can be achieved. On the other hand, it is noticed that in the case of brittle materials, the cyclic fatigue technique which is a traditional pre-cracking method used in metals cannot be adopted to give an ideal pre-crack. Normally, a notch made by a diamond saw is used in the Brazilian disk specimen instead of the ideal crack and a blade-sharpened notch-tip is used instead of the ideal pre-crack. However, experiment results presented by Khan and Al-shayea [5] indicated that the measured mode-I fracture toughness of limestone rock by using straight-notched Brazilian disk specimen is not a material constant, which is dependent on the relative notch length (the ratio of notch length to disk diameter) of the disk specimen.

The main purpose of this paper is to study the pure mode-I fracture behavior of PMMA under quasi-static loading by using Brazilian disk specimen, and investigates the effect of notch shape and relative notch length on the mode-I fracture toughness measurement. The tested material was a commercial PMMA whose density is 1,180 kg/m^3. A diamond saw with a thickness of 0.19 mm was used to make the notch in the center of the disk. Two kinds of notches were made, straight notch and sharp notch, as shown in Figs. 2 and 3. The nominal initial notch lengths for straight-notched specimen and sharp-notched specimen are 11.03 mm and 11.50 mm, respectively. For straight-notched specimen, the disk diameters are 20.00, 23.00 and 25.56 mm, respectively, corresponding to the relative notch length 0.552, 0.480 and 0.432. For sharp-notched specimen, the disk diameters are 19.17, 20.91, 23.00 and 25.56 mm, respectively, corresponding to the relative notch length 0.60, 0.55, 0.50 and 0.45. Therefore, seven different samples were made. The nominal thickness of both straight-notched and sharp-notched specimen is 5 mm. Even though the Brazilian disk specimens had fixed nominal dimensions, the values of disk thickness, disk diameter and notch length were measured independently before each experiment. The Brazilian disk was loaded using a conventional Shimadzu-5000 testing machine at a constant displacement rate of 0.08 mm/min. The experiments were conducted at room temperature.

J. Zhou · Y. Wang (✉) · Y. Xia
CAS Key Laboratory of Mechanical Behavior and Design of Materials, Department of Mechanics and Mechanical Engineering (5th), University of Science and Technology of China, Hefei, Anhui 230026, P.R. China
e-mail: yangwang@ustc.edu.cn

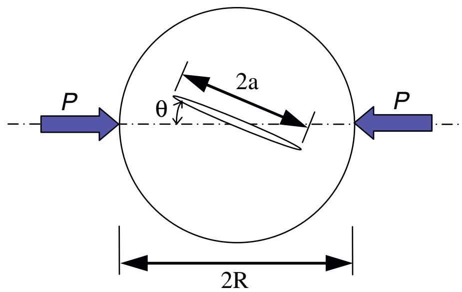

Fig. 1 Schematic diagram of specimen geometry and loading configuration

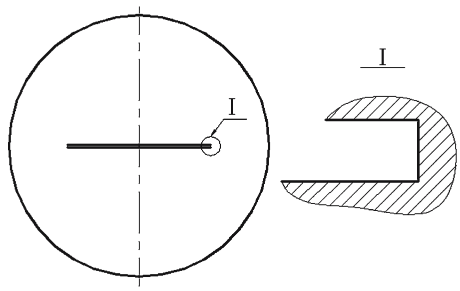

Fig. 2 Front view of Brazilian disk specimen with straight notch

$$K_{\mathrm{I}} = \frac{P}{\pi BR}\sqrt{\pi a}F_{\mathrm{I}} \tag{1}$$

where P is the applied compressive load and B is the disk thickness. F_{I} is a non-dimensional function that only depends on the disk geometry, such as relative crack length, a/R and loading angle, θ.

When the diametrically compressive load P applied to the central cracked circular disk is considered as a concentrated force, F_{I} is given as follows:

$$F_{\mathrm{I}} = f_{11} + 2\sum_{i=1}^{n} A_{1i} f_{1i} \alpha^{2(i-1)} \quad i = 1, 2, \ldots\ldots n \tag{2}$$

where $\alpha = a/R$ is the relative crack length. In Eq. (2),

$$A_{1i}(\theta) = i\cos(2i\theta) - i\cos[2(i-1)\theta] \tag{3}$$

$$f_{1i} = \frac{(2i-3)!!}{(2i-2)!!}\left[1 + \frac{c_{11}}{2i} + \frac{3c_{12}}{4i(i+1)}\right] \tag{4}$$

$$c_{11} = \frac{8 - 4\alpha + 3.8612\alpha^2 - 15.9344\alpha^3 + 24.6076\alpha^4 - 13.234\alpha^5}{\sqrt{1-\alpha}} - 8 \tag{5}$$

$$c_{12} = \frac{-8 + 4\alpha - 0.6488\alpha^2 + 14.1232\alpha^3 - 24.2696\alpha^4 + 12.596\alpha^5}{\sqrt{1-\alpha}} + 8 \tag{6}$$

According to the analysis of Dong et al. [4], the mode-I stress intensity factor at the crack tip in the central cracked circular disk, K_{I}, for an isotropic material, can be expressed by

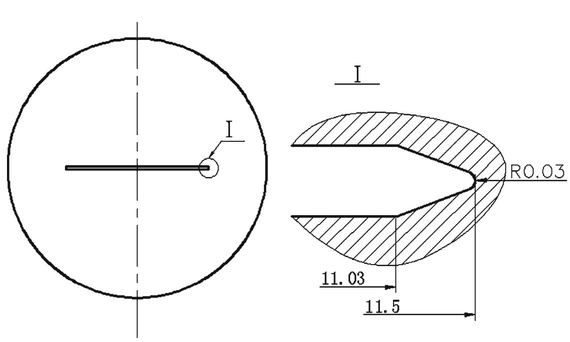

Fig. 3 Front view of Brazilian disk specimen with sharp notch

For central cracked circular disk subjected to pressures, F_{I} is given by

$$F_{\mathrm{I}} = \left[\gamma f_{11}./\sin\gamma + \sum_{i=1}^{n} B_{1i} f_{1i} \alpha^{2(i-1)}/\sin\gamma\right] \tag{7}$$

Here we suppose the total applied load P is distributed symmetrically and uniformly into the pressure over the angle 2γ along the circumference for loading [4]. Now we define the angle between the loading line of total force P and the crack plane is θ_0. In Eq. (7),

$$B_{1i} = \cos(2i\theta_0)\sin(2i\gamma) - \frac{i}{i-1}\cos[2(i-1)\theta_0]\sin[2(i-1)\gamma] \tag{8}$$

It is found that the loading conditions for pure mode-I loading under forces and pressures are $\theta = 0$ and $\theta_0 = 0$

[4]. Such conditions for pure mode-I loading are valid for any values of relative crack length. Consequently, the stress intensity factor for pure mode-I crack can be derived from Eqs. (2) and (7) as follows,

$$K_{\mathrm{I}} = \frac{P}{\pi BR}\sqrt{\pi a} f_{11} \tag{9}$$

$$K_{\mathrm{I}}^{*} = \frac{P}{\pi BR}\sqrt{\pi a}\left\{\gamma f_{11}/\sin\gamma + \sum_{i=1}^{n}\left[\sin(2i\gamma) - \frac{i}{i-1}\sin(2i\gamma - 2\gamma)\right] f_{1i}\alpha^{2(i-1)}/\sin\gamma\right\} \tag{10}$$

where K_{I} and K_{I}^{*} are pure mode-I stress intensity factors corresponding to concentrated force loading and pressure loading, respectively. The critical stress intensity factor (CSIF) of the brittle material under pure mode-I loading conditions can be calculated from the critical external load, $P_{\max}$, at the moment of crack initiation.

In the current investigation, our only concern is the fracture behavior of PMMA under pure mode-I loading conditions. Therefore, the loading angle $\theta = 0$ (or $\theta_0 = 0$) was chosen. Figures 4 and 5 show the typical load–displacement curves for straight-notched and sharp-notched Brazilian disk samples, respectively. It can be seen that for both of two kinds of samples the applied load increases almost linearly before the maximum load is reached and fails quickly at the maximum load. The fracture pattern was observed and it was found that under pure mode-I loading, the crack propagates in a direction parallel to the original crack plane. Therefore, the fracture behavior of PMMA under pure mode-I loading can be characterized as brittle. From these load–displacement curves the maximum load, $P_{\max}$, corresponding to the crack initiation, can be measured and the values of CSIF can be obtained based on Eqs. (9) and (10). For each of seven groups of specimen, several tests were performed and the average values are presented in Figs. 6 and 7.

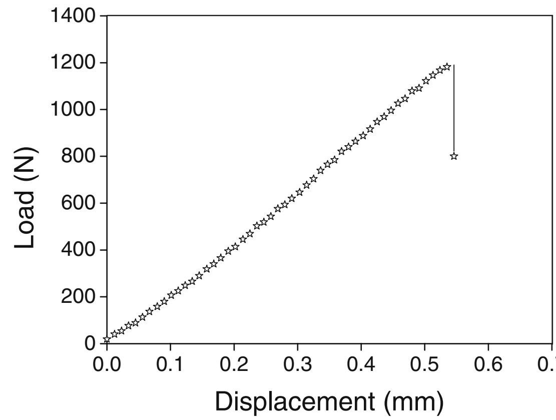

Fig. 5 Load–displacement curve for sharp-notched specimen with $\alpha = 0.45$

Figure 6 shows the mode-I CSIF values of PMMA calculated from Eq. (9) at different relative notch lengths. It is apparent that the measured CSIF is not a constant within the range of investigated relative notch lengths for straight-notched specimen. Its value decreases with increasing relative notch length. However, the CSIF values of sharp-notched specimen can be approximately regarded as a constant within the range of tested specimen geometry. Furthermore, the CSIF values of straight-notched specimen are greater than those of sharp-notched specimen. Such tendency is consistent with the experimental results on alumina and glass presented in literature [6]. It should be emphasized that the fracture toughness is a material constant, which should be not dependent on the relative notch length of the Brazilian disk specimen. The experimental

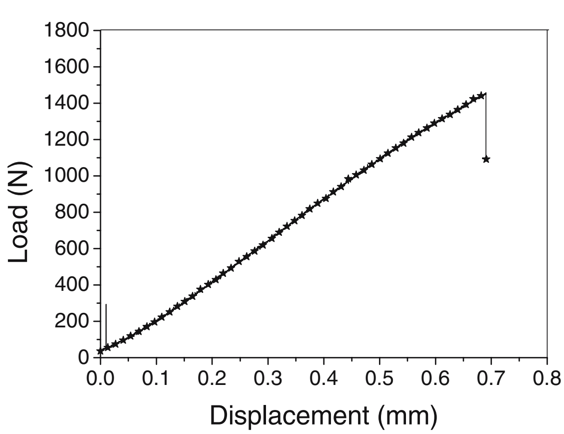

Fig. 4 Load–displacement curve for straight-notched specimen with $\alpha = 0.432$

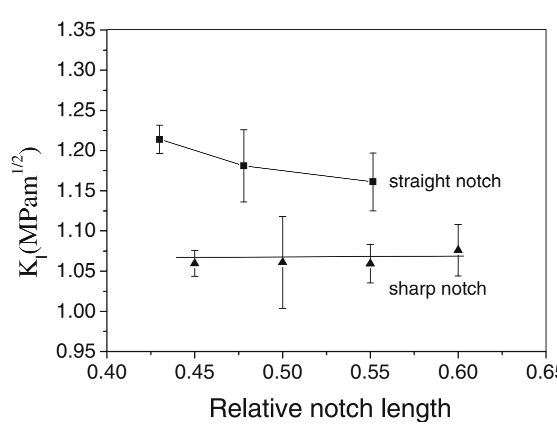

Fig. 6 Measured CSIF calculated from Eq. (9)

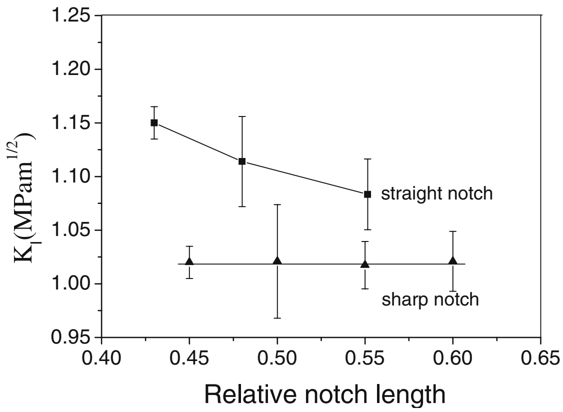

Fig. 7 Measured CSIF calculated from Eq. (10)

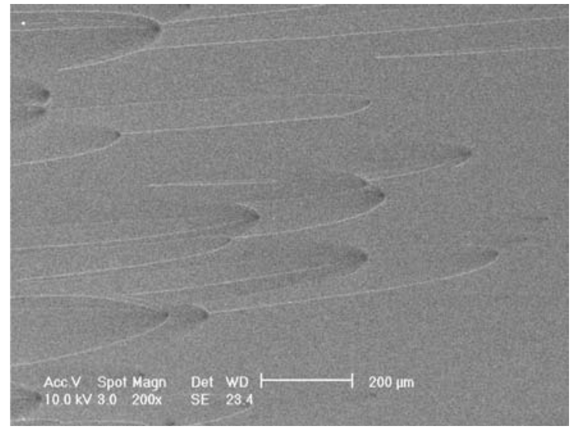

Fig. 8 Morphology of fracture surface of PMMA sharp-notched specimen

results shown in Fig. 6 indicate that the CSIF measured from sharp-notched specimen can be regarded as the fracture toughness of PMMA material.

During each test, a thin carbon paper was placed between the sample and the lower anvil of testing machine to trace the width of the contact area. The load distribution angle is approximately calculated based on this measured width appeared on the thin paper. Therefore, the values of mode-I CSIF incorporating the specimen-anvil contact effect can be calculated from Eq. (10), as seen in Fig. 7. Similarly, the measured CSIF values of straight-notched specimen decrease with the increase of relative notch length and the CSIF values of sharp-notched specimen approximately remain constant over the relative notch length of 0.45–0.6. It is noticed in Figs. 6 and 7 that for both of straight-notched and sharp-notched specimen, there is a decrease in the CSIF values when the CSIF is calculated using pressures instead of concentrated forces. These experimental results indicate that the pressure distribution angle produced due to the specimen deformation cannot be negligible in the fracture toughness measurement. Moreover, it is reasonable to take the CSIF value incorporating contact effect as the intrinsic fracture toughness of PMMA material.

Scanning electron microscope observation on fracture surfaces of PMMA Brazilian disk specimen was conducted on XL30-ESEM, and the typical fractograph is shown in Fig. 8. It is seen that the fracture surface of PMMA under pure mode-I loading is characterized by conic markings. The direction of the apexes of the markings is opposite to the crack-propagation direction. A dark radiative zone is clearly seen inside each conic marking. It has been known that these markings indicate the level differences resulting from an encounter between a microcrack and a main crack during the main crack propagation [7]. It is also found there is no apparent difference on fracture surface morphology between the straight-notched and sharp-notched specimens.

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